Controlling the Population
Adding and Subtracting Polynomials

**Vocabulary**
Match each definition with its corresponding term.

1. polynomial  
   a. a polynomial with only 1 term

2. term  
   b. the degree of the term with the greatest exponent

3. coefficient  
   c. a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients

4. monomial  
   d. a polynomial with exactly 3 terms

5. binomial  
   e. any number being multiplied by a power within a polynomial expression

6. trinomial  
   f. each product in a polynomial expression

7. degree of a term  
   g. a polynomial with exactly 2 terms

8. degree of a polynomial  
   h. the exponent of a term in a polynomial
Problem Set

Identify the terms and coefficients in each expression.

1. $5x + 8$
   The terms are $5x$ and $8$. The coefficients are 5 and 8.

2. $2m^3$

3. $x^2 - 4x$

4. $-3w^4 + w^2 - 9$

5. $-18$

6. $10 - 3x^3 - 6x$

Determine whether each expression is a polynomial. If the expression is not a polynomial, explain why it is not.

7. $9 + 12x$
   The expression is a polynomial.

8. $6m^{\frac{1}{2}}$

9. $\frac{3}{x} - 8x$

10. $-2w^3 + w^2 - 5$

11. $-2.5m$

12. $\frac{x}{7} + 10$

13. $\sqrt{x} + 12$

14. $\frac{4}{5}m - \frac{1}{5}$
Determine whether each polynomial is a monomial, binomial, or trinomial. State the degree of the polynomial.

15. $8x + 3$
   - The polynomial is a binomial with a degree of 1.

16. $5m^2$

17. $x^2 - 7x$

18. $-9n^4 + 6n^2 - 1$

19. $-12$

20. $4 - 10x^3 + 8x$

Write each polynomial in standard form. Classify the polynomial by its number of terms and by its degree.

21. $2x + 6x^2$
   - $6x^2 + 2x$
   - The polynomial is a binomial with a degree of 2.

22. $-9m^2 + 4m^3$

23. $10 - 5x$

24. $7x - 3 + 12x^2$

25. $15 + 4w - w^3$

26. $5x^2 - 15 + 20x$

27. $-1 - p^4$

28. $-6t^2 + 4t + 3t^3$
LESSON 12.1 Skills Practice

29. $-18a^3 + 54a - 22a^2$  
30. $x^3 - x^2 - x^4$

Simplify each expression.

31. $(5x - 8) + (7x + 10)$

$$5x - 8 + 7x + 10$$

$(5x + 7x) + (-8 + 10)$

$$12x + 2$$

32. $(4m^2 + 9m) - (2m^2 + 6)$

33. $(-x^2 + 5x - 12) + (2x^2 - 6)$

34. $(10t^2 - 3t + 9) - (6t^2 - 7t)$

35. $(-5w^2 + 3w - 8) + (15w^2 - 4w + 11)$

36. $(3x^3 + 10x - 1) - (5x^2 + 10x - 9)$

37. $(-a^2 + 2a - 8) + (2a^2 - 9a + 15)$

38. $(14p^4 + 7p^2) + (8p^3 + 7p^2 - p)$

39. $(3x^4 + 3x^3 - 3) - (6x^5 - 9x^3 + 2)$

40. $(-7m^3 - m^2 - m) - (-10m^3 - m - 1)$
The graphs of the functions \( f(x) = 2x + 1 \), \( g(x) = x^2 + x - 3 \), and \( h(x) = f(x) + g(x) \) are shown. Evaluate the function \( h(x) \) for each given value of \( x \). Use the graph of \( h(x) \) to verify your answer.

41. Evaluate \( h(x) \) at \( x = 2 \).

\[
h(x) = f(x) + g(x) \\
= 2x + 1 + x^2 + x - 3 \\
= x^2 + 3x - 2
\]

\[
h(2) = (2)^2 + 3(2) - 2 \\
= 4 + 6 - 2 \\
= 8
\]

42. Evaluate \( h(x) \) at \( x = -4 \).

43. Evaluate \( h(x) \) at \( x = 0 \).

44. Evaluate \( h(x) \) at \( x = 1 \).
45. Evaluate \( h(x) \) at \( x = -2 \).

46. Evaluate \( h(x) \) at \( x = -1.5 \).
They’re Multiplying—Like Polynomials!

Multiplying Polynomials

Problem Set

Determine the product of the binomials using algebra tiles.

1. \((x + 1)\) and \((x + 1)\)

\[(x + 1) (x + 1) = x^2 + 2x + 1\]

2. \((x + 1)\) and \((x + 4)\)

3. \((x + 2)\) and \((x + 2)\)

4. \((x + 3)\) and \((x + 3)\)
5. \(2x + 1\) and \(x + 3\)  

6. \(2x + 3\) and \(x + 2\)

---

Determine the product of the binomials using multiplication tables.

7. \(3x + 4\) and \(2x + 2\)

\[
\begin{array}{ccc}
  \cdot & 2x & 2 \\
3x & 6x^2 & 6x \\
4 & 8x & 8 \\
\end{array}
\]

\((3x + 4)(2x + 2) = 6x^2 + 6x + 8x + 8 = 6x^2 + 14x + 8\)

8. \(5m + 3\) and \(4m + 6\)

9. \(6t + 5\) and \(7t - 5\)

10. \(4x + 2\) and \(4x - 2\)
LESSON 12.2  Skills Practice

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11.  $10w - 1$ and $9w + 8$

12.  $y + 12$ and $5y + 15$

Determine the product of the polynomials using the Distributive Property.

13.  $2x(x + 6)$

\[ 2x(x + 6) = 2x(x) + 2x(6) \]
\[ = 2x^2 + 12x \]

14.  $4x^2(x + 2)$

15.  $7x(x - 5)$

16.  $(2x + 1)(x + 8)$

17.  $(x + 3)(x^2 - 1)$

18.  $(4x + 4)(5x - 5)$

19.  $3x(x^2 + 5x - 1)$

20.  $9x(3x^2 - 4x + 2)$
21. \((x + 2)(x^2 + 6x - 1)\)

22. \((x - 4)(x^2 + 2x - 3)\)
What Factored Into It?
Factoring Polynomials

Vocabulary
State the given property.

1. Symmetric Property of Equality

Problem Set
Factor out the greatest common factor of each polynomial, if possible.

1. $x^2 + 9x$
   $x(x + 9)$

2. $m^2 - 4m$

3. $5x^2 + 20x - 15$

4. $24w^2 - 16$

5. $y^3 - 7y$

6. $2x^3 + 10x^2$

7. $3w + 10$

8. $20x^3 + 16x^2 + 8x$

9. $7m^3 - 21$

10. $15x^3 + 4$
Factor each trinomial using an area model.

11. \( x^2 + 4x + 3 \)

\[
\begin{array}{c|c|c|c}
\hline
 & x & & x + 3 \\
\hline
x & & & \\
\hline
x & x & x & x \\
\hline
x + 1 & & & \\
\hline
1 & & & \\
\hline
\end{array}
\]

\( x^2 + 4x + 3 = (x + 1)(x + 3) \)

12. \( x^2 + 5x + 6 \)

13. \( x^2 - x - 6 \)
14. $x^2 - x - 12$

15. $x^2 + 7x + 10$

16. $x^2 + 3x - 4$
Factor each trinomial completely using multiplication tables. If possible, factor out the greatest common factor first.

17. \(x^2 - 2x - 8\)  
\[
\begin{array}{ccc}
\cdot & x & 2 \\
x & x^2 & 2x \\
-4 & -4x & -8 \\
\end{array}
\]

\(x^2 - 2x - 8 = (x - 4)(x + 2)\)

18. \(y^2 + 13y + 42\)

19. \(m^2 + 6m - 7\)  

20. \(x^2 - 9x + 18\)

21. \(4w^2 + 12w - 40\)  

22. \(2t^2 - 14t^2 + 24t\)

23. \(3m^3 + 36m^2 + 60m\)  

24. \(2x^2 - 8x - 42\)
Factor each polynomial using the trial and error method. If possible, factor out the greatest common factor first.

25. \(x^2 + 11x + 10\)  
The factors of the constant term, 10, are: 
-1, -10  
1, 10  
-2, -5  
2, 5  
\(x^2 + 11x + 10 = (x + 1)(x + 10)\)

26. \(w^2 + 6w - 16\)

27. \(m^2 + 2m - 35\)

28. \(x^2 + 4x - 12\)

29. \(3n^2 - 27n + 60\)

30. \(2x^2 + 22x + 60\)

Factor each polynomial.

31. \(x^2 + 11x + 28 = \frac{(x + 4)(x + 7)}{\text{}}\)
\(x^2 - 11x + 28 = \frac{(x - 4)(x - 7)}{\text{}}\)
\(x^2 + 3x - 28 = \frac{(x - 4)(x + 7)}{\text{}}\)
\(x^2 - 3x + 28 = \frac{(x + 4)(x - 7)}{\text{}}\)

32. \(x^2 + 10x + 9 = \text{}\)
\(x^2 - 10x + 9 = \text{}\)
\(x^2 + 8x - 9 = \text{}\)
\(x^2 - 8x - 9 = \text{}\)

33. \(x^2 + 12x + 27 = \text{}\)
\(x^2 - 12x + 27 = \text{}\)
\(x^2 + 6x - 27 = \text{}\)
\(x^2 - 6x - 27 = \text{}\)

34. \(x^2 + 13x + 40 = \text{}\)
\(x^2 - 13x + 40 = \text{}\)
\(x^2 + 3x - 40 = \text{}\)
\(x^2 - 3x - 40 = \text{}\)
35. $x^2 + 12x + 11 = \underline{\phantom{00}}$
36. $x^2 + 13x + 36 = \underline{\phantom{00}}$

$x^2 - 12x + 11 = \underline{\phantom{00}}$
$x^2 - 13x + 36 = \underline{\phantom{00}}$

$x^2 + 10x - 11 = \underline{\phantom{00}}$
$x^2 + 5x - 36 = \underline{\phantom{00}}$

$x^2 - 10x - 11 = \underline{\phantom{00}}$
$x^2 - 5x - 36 = \underline{\phantom{00}}$

Factor each polynomial completely. If possible, factor out the greatest common factor first.

37. $x^2 + 4x + 4$  
38. $x^2 - 10x + 25$

<table>
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<tr>
<th></th>
<th>$x$</th>
<th>2</th>
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<tbody>
<tr>
<td>$x$</td>
<td>$x^2$</td>
<td>$2x$</td>
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<td>2</td>
<td>2x</td>
<td>4</td>
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$x^2 + 4x + 4 = (x + 2)(x + 2)$

39. $-32 - 12m - m^2$  
40. $45 + 4w - w^2$

41. $5x^2 + 10x - 15$  
42. $4x^2 + 32x + 64$
Zeroing In
Solving Quadratics by Factoring

Vocabulary
Complete the definition of the Zero Product Property.

1. The Zero Product Property states that if the product of two or more factors is equal to

   ________________, then at least one factor must be equal to ________________.

   If \( ab = 0 \), then ________________ or ________________.

   This property is also known as the ________________.

Define the term in your own words.

2. roots

Problem Set
Factor and solve each quadratic equation. Check your answer.

1. \( x^2 + 5x + 6 = 0 \)

   \( x^2 + 5x + 6 = 0 \)

   \( (x + 3)(x + 2) = 0 \)

   Check:

   \( (-3)^2 + 5(-3) + 6 = 0 \)

   \( 9 - 15 + 6 = 0 \)

   \( (-2)^2 + 5(-2) + 6 = 0 \)

   \( 4 - 10 + 6 = 0 \)

   \( x + 3 = 0 \) or \( x + 2 = 0 \)

   \( x = -3 \) or \( x = -2 \)

   The roots are \(-3\) and \(-2\).

2. \( x^2 - 3x - 4 = 0 \)
3. \( m^2 + 2m - 35 = 0 \)

4. \( -x^2 - 4x + 12 = 0 \)

5. \( x^2 + 8x = 0 \)

6. \( w^2 + 50 = -15w \)

7. \( -t^2 + 12t = 32 \)
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8. $x^2 + 2x + 2 = 0$

9. $2t^2 + t - 3 = 0$

10. $w^2 + 5w - 32 = 2w - 4$
Determine the zeros of each quadratic function, if possible. Check your answer.

11. \( f(x) = x^2 - 5x \)
   \[
   f(x) = x^2 - 5x \\
   0 = x^2 - 5x \\
   0 = x(x - 5) \\
   x = 0 \quad \text{or} \quad x - 5 = 0 \\
   x = 0 \quad \text{or} \quad x = 5
   
   \text{The zeros are 0 and 5.}
   
12. \( f(x) = 3x^2 + 6x \)

13. \( f(x) = x^2 + 11x + 30 \)

14. \( f(x) = x^2 - 9x - 36 \)
15. \( f(x) = 2x^2 + 9x + 10 \)

16. \( f(x) = x^2 + 5x + 14 \)

17. \( f(x) = 3x^2 + 3x - 6 \)
18. \( f(x) = \frac{1}{2}x^2 - \frac{3}{4}x \)
What Makes You So Special?
Special Products

Vocabulary
Give an example of each term. Then, factor the expression.

1. perfect square trinomial

2. difference of two squares

3. sum of two cubes

4. difference of two cubes

Problem Set
Factor each binomial completely.

1. $x^2 - 25$
   
   $x^2 - 25 = (x + 5)(x - 5)$

2. $x^3 - 64$

3. $x^3 + 27$

4. $m^2 - 100$
5. \(5x^3 + 40\)  
6. \(t^3 - 125\)

7. \(8a^3 - 27\)  
8. \(x^9 - y^9\)

Factor the trinomial completely.

9. \(x^2 + 16x + 64\)  
   \[x^2 + 16x + 64 = (x + 8)(x + 8)\]

10. \(k^2 - 20k + 100\)

11. \(2x^2 - 28x + 98\)

12. \(5x^2 + 10x + 5\)

13. \(z^3 + 18z^2 + 81z\)

14. \(3x^3 - 30x^2 + 75x\)

Determine the root(s) of each quadratic equation. Check your answer(s).

15. \(x^2 - 100 = 0\)
   
   \[x^2 - 100 = 0\]
   
   \[(x + 10)(x - 10) = 0\]
   
   \[x + 10 = 0\] or \[x - 10 = 0\]
   
   \[x = -10\] or \[x = 10\]
   
   Check:
   
   \((-10)^2 - 100 \neq 0\)  
   \[100 - 100 \neq 0\]
   
   \(10^2 - 100 \neq 0\)  
   \[100 - 100 \neq 0\]
   
   The roots are \(-10\) and \(10\).

16. \(m^2 - 16m + 64 = 0\)
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17. \[6x^2 + 24x + 24 = 0\]

18. \[4x^2 - 9 = 0\]

19. \[t^2 + 22t + 121 = 0\]

20. \[12w^2 - 48w + 48 = 0\]
Determine the zero(s) of each quadratic function. Check your answer(s).

21. \( f(x) = x^2 - 225 \)

\[
f(x) = x^2 - 225 \quad \text{Check:} \\
0 = x^2 - 225 \quad (-15)^2 - 225 \neq 0 \quad (15)^2 - 225 \neq 0 \\
0 = (x + 15)(x - 15) \quad 225 - 225 \neq 0 \quad 225 - 225 \neq 0 \\
x + 15 = 0 \quad \text{or} \quad x - 15 = 0 \\
x = -15 \quad \text{or} \quad x = 15
\]

The zeros are \(-15\) and \(15\).

22. \( f(x) = x^2 + x + \frac{1}{4} \)

23. \( f(x) = 9x^2 - 1 \)
24. \( f(x) = 8x^2 - 48x + 72 \)

25. \( f(x) = 8x^2 - 50 \)

26. \( f(x) = 2x^2 + 52x + 338 \)
Could It Be Groovy to Be a Square?
Approximating and Rewriting Radicals

Vocabulary
Choose the word that best completes each statement.

square root positive (principal) square root radicand
negative square root extract the square root radical expression

1. When solving certain quadratic equations, it is necessary to ________________ from both sides of the equation.
2. Every positive number has both a(n) ________________ and a(n) ________________.
3. The ___________ is the expression enclosed within a radical symbol.
4. A number \( b \) is a(n) ___________ of \( a \) if \( b^2 = a \).
5. An expression involving a radical symbol is called a(n) ________________.

Problem Set
Rewrite each radical by extracting all perfect squares.

1. \( \sqrt{25} \)
   \[ \sqrt{25} = \pm 5 \]
2. \( \sqrt{144} \)
3. \( \sqrt{400} \)
4. \( \sqrt{12} \)
5. \( \sqrt{32} \)
6. \( \sqrt{45} \)
7. \(\sqrt{300}\)  
8. \(5\sqrt{54}\)

Determine the approximate value of each radical expression to the nearest tenth.

9. \(\sqrt{7}\)  
   \[2.6^2 = 6.76\]
   \[2.7^2 = 7.29\]
   \[\sqrt{7} \approx 2.6\]

10. \(\sqrt{37}\)

11. \(\sqrt{96}\)  

12. \(\sqrt{27}\)

13. \(\sqrt{109}\)  

14. \(\sqrt{405}\)

Solve each quadratic equation. Approximate the roots to the nearest tenth.

15. \(x^2 = 40\)
   \[x^2 = 40\]
   \[\sqrt{x^2} = \pm\sqrt{40}\]
   \[x = \pm\sqrt{40}\]
   \[6.3^2 = 39.69\]
   \[6.4^2 = 40.96\]
   \[\sqrt{40} \approx \pm6.3\]
   \[x \approx \pm6.3\]
   The roots are approximately 6.3 and \(-6.3\).
Solve each quadratic equation. Rewrite the roots in radical form.

21. \( x^2 = 48 \)
   - \( x^2 = 48 \)
   - \( \sqrt{x^2} = \pm\sqrt{48} \)
   - \( x = \pm\sqrt{48} \)
   - \( x = \pm\sqrt{16 \cdot 3} \)
   - \( x = \pm\sqrt{16} \cdot \sqrt{3} \)
   - \( x = \pm4\sqrt{3} \)
   - The roots are \( 4\sqrt{3} \) and \( -4\sqrt{3} \).

22. \( x^2 = 52 \)
   - \( x^2 = 52 \)
   - \( \sqrt{x^2} = \pm\sqrt{52} \)
   - \( x = \pm\sqrt{52} \)
   - \( x = \pm\sqrt{4 \cdot 13} \)
   - \( x = \pm\sqrt{4} \cdot \sqrt{13} \)
   - \( x = \pm2\sqrt{13} \)
   - The roots are \( 2\sqrt{13} \) and \( -2\sqrt{13} \).
23. \( x^2 = 27 \)  
24. \( x^2 = 175 \)  

25. \( (12 - x)^2 = 8 \)  
26. \( (x + 20)^2 = 80 \)
Another Method
Completing the Square

Vocabulary
Define the term in your own words.

1. completing the square

Problem Set
Use a geometric figure to complete the square for each expression. Factor the resulting trinomial.

1. \( x^2 + 2x \)

2. \( x^2 + 4x \)

\[
\begin{array}{c|c|c}
1 & x & 1 \\
\hline
x & x^2 & x \\
\hline
x & 1 & \\
\end{array}
\]

\( x^2 + 2x + 1 = (x + 1)^2 \)

3. \( x^2 + 12x \)

4. \( x^2 + 9x \)
Determine the unknown value that would make each trinomial a perfect square.

7. \(x^2 - 10x + \_\) 8. \(x^2 + 14x + \_\)

9. \(x^2 + \_x + 9\) 10. \(x^2 - \_x + 81\)

11. \(x^2 + 7x + \_\) 12. \(x^2 - 15x + \_\)

13. \(x^2 - \_x + 169\) 14. \(x^2 + \_x + \frac{9}{4}\)

Determine the roots of each quadratic equation by completing the square. Round your answer to the nearest hundredth. Check your answer.

15. \(x^2 + 4x - 6 = 0\)

Check:

\[(\pm 1.16)^2 + 4(1.16) - 6 \triangleq 0, \quad (\pm 5.16)^2 - 4(-5.16) - 6 \triangleq 0\]

The roots are approximately 1.16 and -5.16.
16. \( x^2 - 2x - 4 = 0 \)

17. \( x^2 + 10x + 2 = 0 \)

18. \( x^2 - 12x + 25 = 0 \)
19. \( x^2 + 3x - 1 = 0 \)

20. \( x^2 + x - 10 = 0 \)