Did You Find a Parking Space?

Parallel and Perpendicular Lines on the Coordinate Plane

They seem simple enough, but parking lots require a great deal of planning. Transportation engineers use technology and science to plan, design, operate, and manage parking lots for many modes of transportation. During the planning stage of a parking lot, these engineers must keep in mind the needs of the facility that will use the parking lot as well as the needs of the drivers. Engineers must think about the entrances and exits as well as the surrounding streets and their traffic flow. Even the weather must be taken into account if the lot is being built somewhere with heavy rain or snow!

Only thinking about the cars and their drivers, what needs might affect an engineer’s plans? What would make a parking lot “good” or “bad”? Can you think of anything else that might affect the planning of a parking lot other than the factors already mentioned?
Large parking lots have line segments painted to mark the locations where vehicles are supposed to park. The layout of these line segments must be considered carefully so that there is enough room for the vehicles to move and park in the lot without other vehicles being damaged.

The line segments shown model parking spaces in a parking lot. One grid square represents one square meter.

1. What do you notice about the line segments that form the parking spaces?

They are diagonal. They are the same distance apart. They are parallel.

2. What is the vertical distance between $AB$ and $CD$ and between $CD$ and $EF$?

$AB$ or $CD$: 6 meters

$CD$ or $EF$: 6 meters
3. Carefully extend $\overline{AB}$ to create line $p$, extend $\overline{CD}$ to create line $q$, and extend $\overline{EF}$ to create line $r$.

4. Calculate the slope of each line. What do you notice?

$p : m = \frac{3}{2}$  \hspace{1cm} \frac{3 - 0}{2 - 0} \\
$q : m = \frac{3}{2}$ \\
$r : m = \frac{3}{2}$

Remember, the slope is the ratio of the change in the dependent quantity to the change in the independent quantity.
The point-slope form of the equation of a line that passes through \((x_1, y_1)\) and has slope \(m\) is \(y - y_1 = m(x - x_1)\).

5. Use the point-slope form to write the equations of lines \(p\), \(q\), and \(r\). Then, write the equations in slope-intercept form.

\[ p: \left(0, 0\right), m = \frac{3}{2} \]
\[ y - 0 = \frac{3}{2}(x - 0) \]
\[ y = \frac{3}{2}x \]

\[ q: \left(0, 6\right), m = \frac{3}{2} \]
\[ y - 6 = \frac{3}{2}(x - 0) \]
\[ y - 6 = \frac{3}{2}x \]
\[ y = \frac{3}{2}x + 6 \]

\[ r: y = \frac{3}{2}x + 12 \]
6. What do the y-intercepts tell you about the relationship between these lines in the problem situation?

All the lines start at a different point 6 meters above the previous line.
7. If you were to draw $GH$ above $EF$ to form another parking space, predict what the slope and equation of the line will be without graphing the new line. How did you come to this conclusion?

$m = \frac{3}{2} \quad y = \frac{3}{2}x + 18$

$b = 18$
8. Shawna and Lexi made the following statements about parallel lines.

**Shawna**
When you have parallel lines, all of their slopes are going to be equal!

**Lexi**
The y-intercepts of parallel lines are always a multiple of the same number!

a. Explain why Shawna is correct.

If the slopes are not equal, they will intersect.

b. Provide a counter-example to show that Lexi is incorrect.

\[ y = \frac{3}{2}x + 4 \]
\[ y = \frac{3}{2}x + 11 \]
9. Write equations for three lines that are parallel to the line 
\[ y = -2x + 4 \]. Explain how you determined your equations.

1) \[ y = -2x + 6 \] 
2) \[ y = -2x + 8 \] 
3) \[ y = -2x + 10 \]

10. Write an equation for the line that is parallel to the line \( y = 5x + 3 \) and passes through the point \( (4, 0) \). Explain how you determined your equation.

\[ m = 5 \] 
\[ y - 0 = 5(x - 4) \]
\[ y = 5x - 20 \]
11. Without graphing the equations, predict whether the lines given by \( y - 2x = 5 \) and \( 2x - y = 4 \) are parallel.

\[
\begin{align*}
y &= mx + b \\
2x &= 4 + y \\
-4 &= -4 \\
2x - 4 &= y
\end{align*}
\]

\[ y = 2x + 5 \]

Parallel (||)
a. Use the graph to translate line segment $AB$ up $a$ units.

b. Identify the $x$- and $y$-coordinates of each corresponding point on the image.
c. Use the slope formula to calculate the slope of the pre-image.

d. Use the slope formula to calculate the slope of the image.

e. How does the slope of the image compare to the slope of the pre-image?

f. How would you describe the relationship between the graph of the image and the graph of the pre-image?
The line segments shown represent parking spaces in a truck stop parking lot. One grid square represents one square meter.

1. Use a protractor to determine the measures of $\angle VUW$, $\angle XWY$, and $\angle ZYW$. What similarity do you notice about the angles?
2. Carefully extend $\overline{UY}$ to create line $p$, extend $\overline{UV}$ to create line $q$, extend $\overline{WX}$ to create line $r$, and extend $\overline{YZ}$ to create line $s$ on the coordinate plane.
3. Determine whether each set of lines are perpendicular or parallel. Then predict how the slopes of the lines will compare. Do not actually calculate the slopes of the lines when you make your prediction.

a. $q, r, s$
   
   $(\parallel)$ Parallel

b. $p$ and $q$
   
   $(\perp)$ Perp.

c. $p$ and $r$
   
   $(\perp)$ Perp.

d. $p$ and $s$
   
   $(\perp)$ Perp.
4. Calculate the slopes of lines $p$, $xx$, and $s$.

\[
P : \frac{-4}{8} = -\frac{1}{2}
\]
\[
S : \frac{10}{5} = \frac{2}{1}
\]
5. Determine the product of the slopes of two perpendicular lines. Use lines $p$, $q$, $r$, and $s$ to provide an example.
6. Describe the difference between the slopes of two parallel lines and the slopes of two perpendicular lines.

//: Slopes are the same.  
(No intersection) 

\/: Slopes are opposite signs or 
reciprocals.
When the product of two numbers is 1, the numbers are reciprocals of one another. When the product of two numbers is $-1$, the numbers are negative reciprocals of one another. So the slopes of perpendicular lines are negative reciprocals of each other.

7. Do you think that the $y$-intercepts of perpendicular lines tell you anything about the relationship between the perpendicular lines? Explain your reasoning.

No, because it is the SLOPE that tells if a line is $\perp$.

8. Write equations for three lines that are perpendicular to the line $y = -2x + 4$. Explain how you determined your equations.

1) $y = \frac{1}{2}x + 8$
2) $y = \frac{1}{2}x + 6$
3) $y = \frac{1}{2}x + \frac{3}{2}$
9. Write an equation for the line that is perpendicular to the line \( y = 5x + 3 \) and passes through the point \( (4, 0) \). Show all your work and explain how you determined your equation.

\[
m = \frac{-1}{5} \quad \left(4, 0\right) \quad y - y_1 = m(x - x_1)
\]

\[
y - 0 = \frac{-1}{5} (x - 4)
\]

\[
y = \frac{-1}{5}x + \frac{4}{5}
\]

10. Without graphing the equations, determine whether the lines \( y + 2x = 5 \) and \( 2x - y = 4 \) are perpendicular. Explain how you determined your answer.

\[\begin{align*}
y &= -2x + 5 \\
\text{Neither or } &\perp
\end{align*}\]
1. Carefully extend $\overline{GK}$ to create line $p$, extend $\overline{GH}$ to create line $q$, extend $\overline{FJ}$ to create line $r$, and extend $\overline{KL}$ to create line $s$.

2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if $x$ increases by one unit, by how many units does $y$ change?
3. What is the slope of any horizontal line? Does this make sense? Why or why not?

4. Consider the vertical line you drew for Question 1. Suppose that $y$ increases by one unit. By how many units does $x$ change?

5. What is the slope of any vertical line? Does this make sense? Explain why or why not.
6. Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

a. Vertical lines are _________________ parallel.

b. Horizontal lines are _________________ parallel.
7. Describe the relationship between any vertical line and any horizontal line. Explain your reasoning.

8. Write an equation for a horizontal line and an equation for a vertical line that pass through the point \((2, -1)\).
9. Write an equation for a line that is perpendicular to the line $x = 5$ and passes through the point $(1, 0)$.

10. Write an equation for a line that is perpendicular to the line $y = -2$ and passes through the point $(5, 6)$. 
1. Describe the shortest distance between a point and a line.
2. The equation of the line shown on the coordinate plane is \( f(x) = \frac{3}{2}x + 6 \). Draw the shortest segment between the line and the point \( A (0, 12) \). Label the point where the segment intersects \( f(x) \) as point \( B \).

![Graph showing line and point A(0, 12)](image)

3. What information do you need in order to calculate the length of \( \overline{AB} \) using the Distance Formula?
4. How can you calculate the intersection point of \( \overline{AB} \) and the line 
\[ f(x) = \frac{3}{2}x + 6 \] algebraically?
5. Calculate the distance from point A to the line \( f(x) = \frac{3}{2}x + 6 \).
   a. Write an equation for \( \overline{AB} \).
b. Calculate the point of intersection of $\overline{AB}$ and the line $f(x) = \frac{3}{2}x + 6$. 
c. Calculate the length of $\overline{AB}$.

d. What is the distance from point $A$ to the line $f(x) = \frac{3}{2}x + 6$?
Molly’s house is located at point $M (6, 10)$. Jessica’s house is located 2 blocks south and 10 blocks west of Molly’s house at point $J$. Create a graph showing where Molly and Jessica live. Molly and Jessica live the same distance from their school. Use algebra to describe all possible locations of the school.
1. Consider the linear equation $6x - 2y - 5 = 0$.

Decide which of the following lines are parallel, perpendicular, or neither to the given line. Explain your reasoning.

a. $y + 3x = \frac{5}{2}$

b. $12x - 4y = 8$
c. \[ 3y = -x + 5 \]

d. \[ y = \frac{1}{3}x - 5 \]

e. \[ 12x - 10 = 4y \]

f. \[ \frac{2}{3}x + 2y = 8 \]
2. What do you notice about the slopes of the lines perpendicular to the given line?

3. How would you best describe the relationship of all lines perpendicular to the given line?