Whirlygigs for Sale!
Rotating Two-Dimensional Figures through Space

Vocabulary
Describe the term in your own words.

1. disc

Problem Set
Write the name of the solid figure that would result from rotating the plane figure shown around the axis shown.

1. sphere

3.

4.
5. Relate the dimensions of the plane figure to the solid figure that results from its rotation around the given axis.

6. The base of the rectangle is equal to the radius of the cylinder’s base.

7. The base of the rectangle is equal to the radius of the cylinder’s base.

8. The base of the rectangle is equal to the radius of the cylinder’s base.

9. The base of the rectangle is equal to the radius of the cylinder’s base.

10. The base of the rectangle is equal to the radius of the cylinder’s base.
11. 

12. 

Name ____________________________ Date ____________
Cakes and Pancakes
Translating and Stacking Two-Dimensional Figures

Vocabulary
Match each definition to its corresponding term.

1. oblique triangular prism   A. dotted paper used to show three-dimensional diagrams

2. oblique rectangular prism  B. a prism with rectangles as bases whose lateral faces are not perpendicular to those bases

3. oblique cylinder           C. a 3-dimensional object with two parallel, congruent, circular bases, and a lateral face not perpendicular to those bases

4. isometric paper            D. a prism with triangles as bases whose lateral faces are not perpendicular to those bases

5. right triangular prism     E. a 3-dimensional object with two parallel, congruent, circular bases, and a lateral face perpendicular to those bases

6. right rectangular prism    F. a prism with triangles as bases whose lateral faces are perpendicular to those bases

7. right cylinder             G. a prism with rectangles as bases whose lateral faces are perpendicular to those bases
Problem Set

Connect the corresponding vertices of the figure and the translated figure. Name the shape that was translated and name the resulting solid figure.

1. right triangle; right triangular prism

3. 

4. 

5. 

6. 

Name the solid formed by stacking 1000 of the congruent shapes shown.

7. cylinder

8.

9.

10.

11.

12.
Name the solid formed by stacking similar shapes so that each layer of the stack is composed of a slightly smaller shape than the previous layer.

13. cone

14.

15.

16.

17.

18.
Relate the dimensions of the given plane shape to the related solid figure. Tell whether the shape was made by stacking congruent or similar shapes.

19. The lengths of the sides of the triangle are the same as the lengths of the sides of the base of the triangular prism. The triangular prism was made by stacking congruent triangles.

20. 

21. 
LESSON 4.2 Skills Practice

22. 

23. 

24. 

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Cavalieri’s Principles
Application of Cavalieri’s Principles

Vocabulary
Describe the term in your own words.

1. Cavalieri’s principle

Problem Set
Estimate the approximate area or volume each irregular or oblique figure. Round your answers to the nearest tenth, if necessary.

1. The height of each rectangle is 10 yards and the base of each rectangle is 1.5 yards.

I determined that the area is approximately 300 square yards.

area = base × height
= (1.5 × 20)(10)
= (30)(10)
= 300

I determined the sum of the areas of the rectangles to estimate the area of the figure because Cavalieri’s principle says that the area of the irregular figure is equal to the sum of the areas of the multiple rectangles when the base and height of all the rectangles are equal.
2. The height of each rectangle is 0.6 inch and the base of each rectangle is 2 inches.
Spin to Win
Volume of Cones and Pyramids

Problem Set

Calculate the volume of each cone. Use 3.14 for $\pi$.

1. [Diagram of a cone with dimensions 5 cm and 4 cm]

   \[
   \text{volume} = \frac{1}{3} Bh \\
   = \frac{1}{3} \pi r^2 h \\
   = \frac{1}{3} \pi (4)^2 (5) \\
   \approx 83.73 \text{ cubic centimeters}
   \]

2. [Diagram of a cone with dimensions 2 cm and 7 cm]
3. \[ \text{6 in.} \quad \text{3 in.} \]

4. \[ \text{4 in.} \quad \text{13 in.} \]

5. \[ \text{10 m} \quad \text{15 m} \]
LESSON 4.4 Skills Practice

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6. 

7. 

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8. 

9. 

10.
Calculate the volume of the square pyramid.

11.

\[ \text{Volume} = \frac{1}{3} Bh \]
\[ = \frac{1}{3} s^2h \]
\[ = \frac{1}{3} (10)^2(9) \]
\[ = 300 \text{ cubic inches} \]

12.
13. [Diagram of a pyramid with dimensions: 11 cm (height), 7 cm (base side), 7 cm (base side)]

14. [Diagram of a pyramid with dimensions: 20 m (height), 25 m (slant height), 25 m (base side)]
LESSON 4.4 Skills Practice

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15. 

![Diagram of a pyramid with dimensions: 22 ft at the base, 30 ft on each side, and 30 ft in height.]

16. 

![Diagram of another pyramid with dimensions: 28 mm at the base, 21 mm on each side, and 21 mm in height.]

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Chapter 4 Skills Practice  437
17. 

34.5 in. 

42 in. 

42 in. 

18. 

75 cm 

90 cm 

90 cm
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19.  

![Diagram of a pyramid with dimensions 125 yd, 100 yd, and 100 yd.]  

20.  

![Diagram of a pyramid with dimensions 180 ft, 200 ft, and 200 ft.]
Vocabulary

Describe a similarity and a difference between each term.

1. radius of a sphere and diameter of a sphere
2. cross section of a sphere and great circle of a sphere
3. hemisphere and sphere

Describe the term in your own words.

4. annulus
Problem Set

Calculate the volume of each sphere. Use 3.14 for $\pi$. Round decimals to the nearest tenth, if necessary.

1. $r = 7$ meters

   $r$

   Volume $= \frac{4}{3} \pi r^3$
   
   $= \frac{4}{3} \pi (7)^3$
   
   $= \frac{1372}{3} \pi$
   
   $\approx 1436.0$ cubic meters

2. $r = 6$ inches

   $r$

   $d = 20$ inches

   $d$

3. $d = 20$ inches

4. $d = 16$ meters

5. $r = 2.5$ centimeters

   $r$

6. $r = 11.25$ millimeters

   $r$
LESSON 4.5 Skills Practice

7. The radius of the great circle of a sphere is 8 meters.

8. The radius of the great circle of a sphere is 12 feet.

9. The diameter of the great circle of a sphere is 20 centimeters.

10. The diameter of the great circle of a sphere is 15 yards.
Turn Up the . . .
Using Volume Formulas

Problem Set
Calculate the volume of each pyramid.

1. 
   \[ V = \frac{1}{3} Bh \]
   \[ = \frac{1}{3} (3)(3)(5) \]
   \[ = 15 \text{ cubic meters} \]

3. 

4. 

Calculate the volume of each cylinder. Use 3.14 for \( \pi \). Round decimals to the nearest tenth, if necessary.

7. \( V = \pi r^2 h \)
   \[ = \pi (5.5)^2 (7) \]
   \[ = 211.75 \pi \]
   \[ \approx 664.9 \text{ cubic meters} \]

8. \[ V = \pi r^2 h \]
   \[ = \pi (30)^2 (22) \]
   \[ = 6300 \pi \]
   \[ \approx 19851.9 \text{ cubic yards} \]
Name ___________________________ Date ________

11. 4 mm

12. 16 ft

13. 9 m

14. 3.5 cm
Calculate the volume of each cone. Use 3.14 for \( \pi \). Round decimals to the nearest tenth, if necessary.

15. \( \overline{h} = 6 \text{ mm} \), \( r = 5 \text{ mm} \)

\[
V = \frac{1}{3} \pi r^2h \\
= \frac{1}{3} \pi (5)^2(6) \\
= 50\pi \\
\approx 157 \text{ cubic millimeters}
\]

16. \( 12 \text{ m} \), \( 20 \text{ m} \)

\[
V = \frac{1}{3} \pi r^2h \\
= \frac{1}{3} \pi (12)^2(20) \\
\approx \text{volume}
\]

17. \( 10 \text{ ft} \), \( 7 \text{ ft} \)

\[
V = \frac{1}{3} \pi r^2h \\
= \frac{1}{3} \pi (7)^2(10) \\
\approx \text{volume}
\]

18. \( 11 \text{ yd} \), \( 4 \text{ yd} \)

\[
V = \frac{1}{3} \pi r^2h \\
= \frac{1}{3} \pi (4)^2(11) \\
\approx \text{volume}
\]
LESSON 4.6  Skills Practice

19.  
\[ \text{ cone base radius: } 8 \text{ m} \]
\[ \text{ cone height: } 6 \text{ m} \]

20.  
\[ \text{ cone base radius: } 3 \text{ ft} \]
\[ \text{ cone height: } 3 \text{ ft} \]

21.  
\[ \text{ cone base radius: } 3 \text{ in.} \]
\[ \text{ cone height: } 8 \text{ in.} \]

22.  
\[ \text{ cone base radius: } 2 \text{ mm} \]
\[ \text{ cone height: } 5 \text{ mm} \]
Calculate the volume of each sphere. Use 3.14 for $\pi$. Round decimals to the nearest tenth, if necessary.

23. 

$V = \frac{4}{3} \pi r^3$

$= \frac{4}{3} \pi (9)^3$

$= 972\pi$

$\approx 3052.08$ cubic inches

24. 

$V = \frac{4}{3} \pi r^3$

$= \frac{4}{3} \pi (10)^3$

$= 4186\pi$

$\approx 13355.45$ cubic centimeters
25. 14 mm

26. 2.5 ft
Tree Rings
Cross Sections

Problem Set
Describe the shape of each cross section shown.

1. The cross section is a rectangle.

3.

4.
Use the given information to sketch and describe the cross sections.

11. Consider a cylinder. Sketch and describe three different cross sections formed when a plane intersects a cylinder.

![Cross sections: circle, rectangle, ellipse](image)

12. Consider a rectangular prism. Sketch and describe three different cross sections formed when a plane intersects a rectangular prism.

13. Consider a pentagonal prism. Sketch and describe three different cross sections formed when a plane intersects a pentagonal prism.
14. Consider a triangular prism. Sketch and describe three different cross sections formed when a plane intersects a triangular prism.

15. Consider a triangular pyramid. Sketch and describe three different cross sections formed when a plane intersects a triangular pyramid.

16. Consider a hexagonal pyramid. Sketch and describe three different cross sections formed when a plane intersects a hexagonal pyramid.
Consider two cross sections of the given solid. One cross section is parallel to the base of the solid, and the other cross section is perpendicular to the base of the solid. Determine the shape of each of these cross sections.

17. A cross section that is parallel to the base is a hexagon congruent to the hexagonal bases. A cross section that is perpendicular to the base is a rectangle.

18. [Diagram of a solid with cross sections indicated]

19. [Diagram of a solid with cross sections indicated]
20.

21.

22.
LESSON 4.7  Skills Practice

23.

24.

Draw a solid that could have each cross section described.

25. cross section parallel to the base

The solid is a cone. (The solid could also be a cylinder.)
26. cross section perpendicular to the base

27. cross section parallel to the base

28. cross section parallel to the base
LESSON 4.7  Skills Practice

Name ____________________________________________  Date ____________

29. cross section perpendicular to the base

30. cross section parallel to the base
Two Dimensions Meet Three Dimensions
Diagonals in Three Dimensions

Problem Set

Draw all of the sides you cannot see in each rectangular solid using dotted lines. Then, draw a three-dimensional diagonal using a solid line.

1. 

2. 

3. 

4. 

5. 

6.
Determine the length of the diagonal of each rectangular solid.

7. 

The length of the diagonal of the rectangular solid is about 12.33 inches.

The length of the first leg is 10 inches.

Length of Second Leg: 
\[ d^2 = 6^2 + 4^2 \]
\[ = 36 + 16 \]
\[ = 52 \]
\[ d = \sqrt{52} \approx 7.21 \]

Length of Diagonal: 
\[ d = 7.21^2 + 10^2 \]
\[ = 51.98 + 100 \]
\[ d = \sqrt{151.98} \approx 12.33 \]
9. A triangular prism with dimensions of 15 cm, 10 cm, and 6 cm.

10. A right rectangular prism with dimensions of 7 yd, 5 yd, and 7 yd.
11. 

![Diagram of a solid figure with dimensions 3" x 15° x 5".]

12. 

![Diagram of a solid figure with dimensions 2 ft x 2 ft x 12 ft.]

4
Diagonals are shown on the front panel, side panel, and top panel of each rectangular solid. Sketch three triangles using the diagonals from each of the three panels and some combination of the length, width, and height of the solid.

13.

14.
15. 

```
4 ft

5 ft

6 ft

L

W

H
```

16. 

```
9 cm

5 cm

8 cm

L

W

H
```

17. 

```
8 yd

10 yd

4 yd

L

W

H
```
A rectangular solid is shown. Use the diagonals across the front panel, the side panel, and the top panel of each solid to determine the length of the three-dimensional diagonal.

18. 

\[ SD^2 = \frac{1}{2}(8^2 + 6^2 + 3^2) \]
\[ SD^2 = \frac{1}{2}(64 + 36 + 9) \]
\[ SD^2 = \frac{1}{2}(109) \]
\[ SD^2 = 54.5 \]
\[ SD = \sqrt{54.5} \approx 7.4 \]

The length of the three-dimensional diagonal is \( \sqrt{54.5} \) or approximately 7.4 inches.
20. A rectangular prism with dimensions 3 m x 9 m x 10 m.

21. A rectangular prism with dimensions 8 ft x 10 ft x 12 ft.
LESSON 4.8 Skills Practice

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22. 

23. 

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Use a formula to answer each question. Show your work and explain your reasoning.

25. A packing company is in the planning stages of creating a box that includes a diagonal support. The box has a width of 5 feet, a length of 6 feet, and a height of 8 feet. How long will the diagonal support need to be?

The diagonal support should be approximately 11.18 feet. I determined the answer by calculating the length of the box’s diagonal.

\[ d^2 = 5^2 + 6^2 + 8^2 \]
\[ d^2 = 25 + 36 + 64 \]
\[ d^2 = 125 \]
\[ d = 11.18 \]
26. A plumber needs to transport a 12-foot pipe to a jobsite. The interior of his van is 90 inches in length, 40 inches in width, and 40 inches in height. Will the pipe fit inside the plumber’s van?

27. You are landscaping the flower beds in your front yard. You choose to plant a tree that measures 5 feet from the root ball to the top. The interior of your car is 60 inches in length, 45 inches in width, and 40 inches in height. Will the tree fit inside your car?

28. Julian is constructing a box for actors to stand on during a school play. To make the box stronger, he decides to include diagonals on all sides of the box and a three-dimensional diagonal through the center of the box. The diagonals across the front and back of the box are each 2 feet, the diagonals across the sides of the box are each 3 feet, and the diagonals across the top and bottom of the box are each 7 feet. How long is the diagonal through the center of the box?
29. Carmen has a cardboard box. The length of the diagonal across the front of the box is 9 inches. The length of the diagonal across the side of the box is 7 inches. The length of the diagonal across the top of the box is 5 inches. Carmen wants to place a 10-inch stick into the box and be able to close the lid. Will the stick fit inside the box?

30. A technician needs to pack a television in a cardboard box. The length of the diagonal across the front of the box is 17 inches. The length of the diagonal across the side of the box is 19 inches. The length of the diagonal across the top of the box is 20 inches. The three-dimensional diagonal of the television is 24 inches. Will the television fit in the box?