

3.5 - Solving Matrix Equations

We can solve systems of equations using matrices

Given the system: $3x - 6y = 9$

$$5x + 2y = 9$$

Write the coefficients in one matrix with the x's in the first column and the y's in the second column

$$\begin{bmatrix} 3 & -6 \\ 5 & 2 \end{bmatrix}$$

Write the variables in another matrix

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

Finally write the numbers in another matrix

$$\begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

Organize your three matrices so they have the coefficients first, the variables second and set that equal to the numbers.

$$\begin{bmatrix} 3 & -6 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

$3(2) - (-6)(5)$
 $6 + (+30) = 36$
 $\frac{1}{36} \begin{bmatrix} 2 & 6 \\ -5 & 3 \end{bmatrix}$

Next, find the inverse of the coefficient matrix and write the inverse in front of the coefficient matrix and in front of the number matrix

~~$$\frac{1}{36} \begin{bmatrix} 2 & 6 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 2 & 6 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$~~

The matrices on the left hand side cancel to leave just the variable matrix and the matrices on the right hand side you just multiply.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 2 & 6 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

2×2 2×1

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 72 \\ -18 \end{bmatrix} = \begin{bmatrix} 72 \\ -18 \end{bmatrix} \begin{matrix} \div 36 \\ \div 36 \\ \div 36 \end{matrix} = \begin{bmatrix} 2 \\ -1/2 \end{bmatrix}$$

$$\begin{bmatrix} 2(9) + 6(9) \\ -5(9) + 3(9) \end{bmatrix}$$

Ex $12x + 5y = 30$
 $4x - y = 6$

$$\begin{bmatrix} 12 & 5 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ 6 \end{bmatrix}$$

$$12(-1) - 5(4) = -12 - 20 = -32$$

~~$$\begin{bmatrix} 1 & -5 \\ -4 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-32} \begin{bmatrix} -1 & -5 \\ -4 & 12 \end{bmatrix} \begin{bmatrix} 30 \\ 6 \end{bmatrix}$$~~

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-32} \begin{bmatrix} 1 & -5 \\ 4 & 12 \end{bmatrix} \begin{bmatrix} 30 \\ 6 \end{bmatrix} = \frac{1}{-32} \begin{bmatrix} -60 \\ 48 \end{bmatrix} = \begin{bmatrix} \frac{-60}{-32} \\ \frac{48}{-32} \end{bmatrix}$$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \text{value} \\ \text{value} \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$\begin{aligned} &1(-1) - 1(1) \\ &-1 - 1 = -2 \end{aligned}$$

~~$$\frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$~~

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -8 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$4y = 6x - 12$$

$$5x = 2y + 10$$

$$\begin{bmatrix} -6 & 4 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 10 \end{bmatrix}$$

$$-6(-2) - 4(5) = 12 - 20 = -8$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-8} \begin{bmatrix} -2 & -4 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} -12 \\ 10 \end{bmatrix} = \frac{1}{-8} \begin{bmatrix} -16 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} -2(-12) + -4(10) \\ 24 + -40 \\ -5(-12) + -6(10) \\ 60 + -60 \end{pmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\textcircled{10} \begin{bmatrix} -6 & 11 \\ -4 & 7 \end{bmatrix} \quad -6(7) - 11(-4) \\ -42 + (+44) = 2$$

$$\frac{1}{2} \begin{bmatrix} 7 & -11 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} & -\frac{11}{2} \\ 2 & 3 \end{bmatrix}$$

②
$$\begin{aligned} -2x + 5y &= -11 \\ -x &= -3y + 8 \end{aligned}$$

$$\begin{aligned} -2x + 5y &= -11 \\ x - 3y &= 8 \end{aligned}$$

$$\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 8 \end{bmatrix}$$