3.4 - Determinants and Inverses of Matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

To find the determinant of a 2x2 matrix use the following formula: ad-bc

Find the determinant of the following matrices:

$$\begin{vmatrix} 4 & -2 \\ 5 & 6 \end{vmatrix} = 4(6)-(-2)5 = 24-(-10) = 34$$

$$\begin{vmatrix} 5 & 0 \\ 6 & -3 \end{vmatrix} = 5(-3)-0(6) = -15$$

$$\begin{vmatrix} 8 & 3 \\ 8 & 3 \end{vmatrix} = 8(3)-3(8)$$

$$24-24 = \boxed{0}$$

Inverses of 2x2 Matrices

Find the determinant first. If the determinant is not zero, then go on to the following steps, if the determinant is 0, then there is no inverse.

Determinant of 0 = No inverse

Start with: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ first switch a and d to get $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$

then change the sign of b and c to get $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

After rearranging your matrix, multiply each number in your matrix by 1 over the determinant.

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples

$$\begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$$

$-3(1) - 2(-2)$

$-3 - (-4)$

$= 1$

$$\frac{1}{1}\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ 8 & 6 \end{bmatrix} \quad 4(6) - 3(8)$$

$$24 - 24 = 0$$

No inverse

$$\begin{bmatrix} 1 & 6 \\ 1 & 5 \end{bmatrix} \quad 1(5) - 6(1)$$

$$5 - 6 = -1$$

$$\frac{1}{-1}\begin{bmatrix} 5 & -6 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 \\ 2 & 7 \end{bmatrix} \quad 3(7) - 4(2)$$

$$21 - 8 = 13$$

$$\frac{1}{13}\begin{bmatrix} 7 & -4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{7}{13} & \frac{-4}{13} \\ \frac{-2}{13} & \frac{3}{13} \end{bmatrix}$$

$$\begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}$$

$5(2)-6(3)$

$10-18=-8$

$$\frac{1}{-8}\begin{bmatrix} 2 & -6 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} \frac{2}{-8} & \frac{-6}{-8} \\ \frac{-3}{-8} & \frac{5}{-8} \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} \frac{1}{-4} & \frac{3}{4} \\ \frac{3}{8} & \frac{5}{-8} \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 6 & -8 \end{bmatrix}_{3\times 2} \begin{bmatrix} -11 & 12 & 0 \\ 10 & 4 & 7 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 9 & 20 & 14 \\ 40 & 16 & 28 \\ -146 & 40 & -56 \end{bmatrix}$$

$$\begin{bmatrix} 1(-11)+2(10) & 1(12)+2(4) & 1(0)+2(7) \\ \cancel{0(-11)}+4(10) & \cancel{0(12)}+4(4) & 0(0)+4(7) \\ 6(-11)+-8(10) & 6(12)+-8(4) & 6(0)+-8(7) \end{bmatrix}$$

$+$ or $-$

+/- corresponding elements

same dimensions

$\times$

$a \times \underline{b} \quad \underline{b} \times c$

$= a \times c$