

### 3.3 - Matrix Multiplication

Matrix Multiplication depends on dimensions as well.

The number of columns in the first matrix must match the number of rows in the second matrix.

$3 \times 4 \cdot 4 \times 1$  - So you could multiply a  $3 \times 4$  matrix with a  $4 \times 1$  matrix  
 ~~$4 \times 1 \cdot 3 \times 4$~~  - You could not multiply a  $3 \times 4$  matrix with a  $3 \times 4$  matrix

The matrix you will get after multiplying will have the following dimensions:

-The number of rows from the first matrix by the number of columns of the second.

So back to our first example: Multiplying a  $3 \times 4$  with a  $4 \times 1$  will get you a  $3 \times 1$  matrix

Order matters when doing matrix multiplication!!

$A(B) \neq B(A)$

Can I Multiply the following matrices together?

$2 \times \cancel{2} \text{ \& } \cancel{2} \times 7$

Yes

New Dimensions

$2 \times 7$

$1 \times \cancel{5} \text{ \& } \cancel{5} \times 3$

Yes

$1 \times 3$

$4 \times \underline{1} \text{ \& } \underline{4} \times 1$

No

$$A = \begin{bmatrix} 2 & 0 \\ -1 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ -2 & -3 \\ 4 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 6 & -4 \\ -1 & 12 \\ -2 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} -3 & 1 & 4 \\ -2 & 2 & 6 \end{bmatrix}$$

### 3D — Scalar Multiplication

Multiply every element in the matrix by the scalar.

$$3D = \begin{bmatrix} -9 & 3 & 12 \\ -6 & 6 & 18 \end{bmatrix}$$

$-6A$

$$\begin{bmatrix} 2 & 0 \\ -1 & -5 \end{bmatrix} = A$$

$$-6A = \begin{bmatrix} -12 & 0 \\ 6 & 30 \end{bmatrix}$$

$AB$  - N.P.  
 $2 \times 2$   $3 \times 2$

$BD$  -  $3 \times 3$   
 $3 \times 2$   $2 \times 3$

$$\begin{bmatrix} 1 & 3 \\ 2 & -3 \\ 4 & 0 \end{bmatrix}^{3 \times 2} \begin{bmatrix} -3 & 1 & 4 \\ -2 & 2 & 6 \end{bmatrix}^{2 \times 3} = \begin{bmatrix} -9 & 7 & 22 \\ 12 & -8 & -26 \\ -12 & 4 & 16 \end{bmatrix}^{3 \times 3}$$

$1(-3) + 3(-2)$ $-3 + -6$	$1(1) + 3(2)$ $1 + 6$	$1(4) + 3(6)$ $4 + 18$
$-2(-3) + -3(-2)$ $6 + 6$	$-2(1) + -3(2)$ $-2 + -6$	$-2(4) + -3(6)$ $-8 + -18$
$4(-3) + 0(-2)$ $-12 + 0$	$4(1) + 0(2)$ $4 + 0$	$4(4) + 0(6)$ $16 + 0$

$$\begin{bmatrix} \cancel{2} & \cancel{6} \\ 8 & -3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 32 \\ 32 & -7 \end{bmatrix}$$

$2 \times 2$                    $2 \times 2$                    $2 \times 2$

$$\begin{bmatrix} 2(4) + 6(0) & 2(1) + 6(5) \\ 8(4) + -3(0) & 8(1) + -3(5) \end{bmatrix}$$

$$\begin{bmatrix} \cancel{-6} \\ \cancel{-5} \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} 7 & 12 \end{bmatrix}_{1 \times 2} = \begin{bmatrix} -42 & -72 \\ -35 & -60 \\ 21 & 36 \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} -6(7) & -6(12) \\ -5(7) & -5(12) \\ 3(7) & 3(12) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 7 & 8 \\ -2 & 6 & 0 & 1 \end{bmatrix}_{2 \times 4} \begin{bmatrix} 11 & 0 \\ 8 & 7 \\ 6 & 2 \\ 3 & 1 \end{bmatrix}_{4 \times 2} = \begin{bmatrix} 109 & 50 \\ 29 & 43 \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 1(11) + 4(8) + 7(6) + 8(3) & 1(0) + 4(7) + 7(2) + 8(1) \\ 11 + 32 + 42 + 24 & 0 + 28 + 14 + 8 \\ -22 + 48 + 0 + 3 & -2(0) + 6(7) + 0(2) + 1(1) \\ & 0 + 42 + 0 + 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{2 \times 2} =$$

$$\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 1(5) + 2(7) \\ 5 + 14 \\ 3(5) + 4(7) \\ 15 + 28 \end{bmatrix}$$

$$\begin{bmatrix} 1(6) + 2(8) \\ 6 + 16 \\ 3(6) + 4(8) \\ 18 + 32 \end{bmatrix}$$